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INVESTIGATIONS ON LIGHT AND HEAT, MADE AND PUBLISHED WHOLLY OR IN PART WITH APPROPRIATION FROM THE RUMFORD FUND.

XV.

ON THE THERMAL CONDUCTIVITY OF MILD STEEL.

BY EDWIN H. HALL.

Presented January 8, 1896.

SOME years ago,* the author gave an account of certain experiments which he had made, by the method of Forbes, upon the thermal conductivity of cast iron and cast nickel, and stated that he hoped "soon to make trial of a method for estimating thermal conductivities somewhat different from any heretofore used." This method has now been tested, and it appears to be a good one, although certain changes in the apparatus used, and presently to be described, might be made with advantage.

It is well known that early experiments upon thermal conductivity were made with thin plates or sheets of metal. The results obtained were absurdly erroneous, owing mainly to the difficulty of determining accurately the difference in temperature of the two faces of the plates. Later the method was by various devices made to give better results, but none satisfactory, and for a long time it has been discarded, replaced generally by the method of Forbes, steady flow of heat in a long bar exposed to the air, or that of Angström, waves of heat sent along the bar at regular intervals. None of the methods have been very simple, or easy of application, and the results obtained in the case of the metal most studied, iron, have been as various as the difficulty of the investigation would lead one to expect.

Every one who has engaged in the experimental study of heat conduction, or has examined the curiously indirect methods employed in this study, must have regretted the apparent impossibility of using successfully the old method of thin plates. About four years ago, it occurred to the author that one might determine the difference in temperature of the two faces of a thin plate by making these faces the junctions of a thermo-electric element. Suppose, for example, that a plate of iron is coated on its two broad faces with a layer of copper, and that copper wires lead from these copper sheets to a galvanometer. If one copper side is exposed to water at t°, and the other copper side to water at t'°, heat will flow through the whole plate, and the difference in temperature of the two surfaces where the iron joins the copper sheets will give rise to a thermo-electric current, which may be made to give an accurate measure of the difference of temperature of the two faces of the iron plate. No attempt need be made to determine the difference in temperature of the outer surfaces of the copper sheets. This is an outline of the most distinctive feature of the method now to be described. The chief obstacle has been, and still is, the difficulty of getting a sufficiently even temperature over the whole of each surface.

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* These Proceedings, Vol. XXVII. p. 262, 1892.

Figure 1 shows with tolerable accuracy a vertical section through the conducting disk and the apparatus most closely connected with it, the parts being represented at one third their actual size. Certain unessential changes have been made in the drawing, in order to illustrate as many features as possible in this one figure.

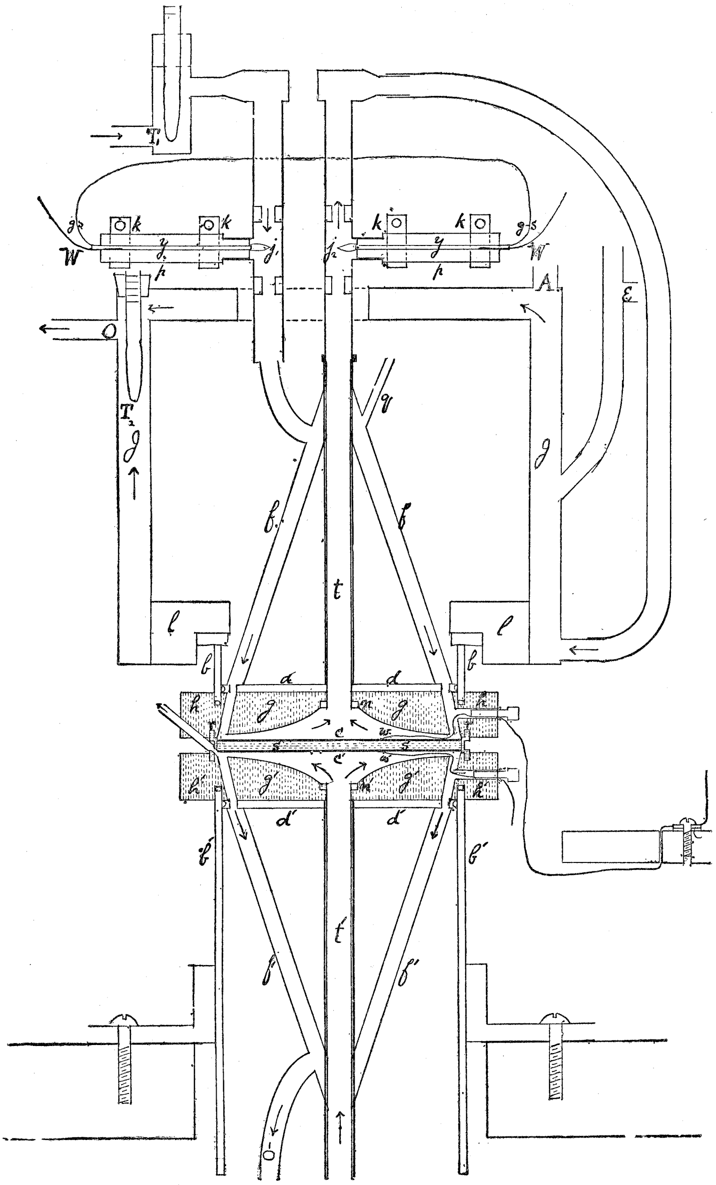
S is the conducting disk of iron, or, rather, "open-hearth" boiler-plate steel. The following information concerning its quality and composition was furnished by Mr. Parker, of Parker, Field, and Mitchell, Cambridgeport, who made the disk: —

Tensile strength	62,300 lbs. per sq. in.
Elastic limit	34,800 " "
Phosphorus	.0035.
Carbon	.001.

The analysis was made by the Central Iron and Steel Works of Harrisburg. The metal is very soft and tough, as the composition would indicate.

The mean thickness of the disk, which was carefully measured at many points, is 0.295 cm., the smallest thickness found being 0.290 cm., and the largest, 0.299 cm. The diameter of the disk is 9.80 cm.

The parts *c* and *c'* are the coatings of copper deposited electrolytically upon the steel. The author hesitated at first to apply copper in this way, fearing that galvanoplastic copper might be seriously different in thermo-electric quality from ordinary copper wire, which must be attached to the coatings; but all his own experience and that of Professor B. O. Peirce, who has had occasion to


T₁
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k
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g
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y
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W
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j₂
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A
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Fig. 1.

investigate the same matter recently, go to show that this apprehension was groundless, that different specimens of copper wire now purchased in the American market differ very little among themselves in thermo-electric quality, and differ very little from galvanoplastic copper.

The ordinary method of depositing copper, from a sulphate solution, is not immediately available for a steel plate, because the solution attacks steel. On the other hand, the method which uses a cyanide solution is comparatively slow, and requires much time for a thick coating. Accordingly, the disk here used was subjected to the depositing current in a cyanide solution for a sufficient time to cover the surface completely, and was then transferred to a sulphate solution, where the process was continued for many hours, until the coating was about 0.05 cm. thick on each side of the disk.

The letters *r r* in the Figure mark a ring of steel, similar in quality to the disk. It was turned a bit too small to receive the disk when both were at the same temperature. The ring was then heated, and the disk, smeared on the edge with a pitchy cement, was slipped into it. The contraction of the ring in cooling made a joint so tight that under severe tests no leakage through it has been detected.

The parts *h h* and *h′h′* are rings of hard rubber. These rings do not extend quite to the copper coatings of the disk, but leave a line of the steel ring *r r*, about 0.07 cm. wide exposed, above and below these coatings. These exposed edges have caused trouble through rusting, under the action of the water flowing through the apparatus. No varnish protects them effectually. In future experiments care will be taken to have no iron or steel similarly exposed.

The parts *b b* and *b′b′* are hollow brass cylinders about 10 cm. in external diameter, fitting into grooves in the hard rubber rings *h h* and *h′h′* respectively, a small rubber tube at the bottom of each groove being used to make, under pressure, a water-tight packing of the joint. At first these brass rings fitted directly into the ring *r r*, the rings *h h* and *h′h′* not being used. But it soon appeared that heat, carried to or from the ring *r r* by the rings *b b* and *b′b′*, each of which in the first arrangement exposed a broad surface to the water, made the difference in temperature between the top and bottom of the disk much greater near the edge than elsewhere. For it is to be noted that the temperature difference between top and bottom of the disk was in most parts only a small fraction, perhaps 5%, of the maximum temperature difference between the water above and the water below the disk.

The parts *d d* and *d′d′* are disks of brass, the edges of which press upon the rings *h h* and *h′h′*. These disks are perforated near the edge by numerous holes for flow of water. The disks are packed water-tight, or nearly so, in the brass rings *b b* and *b′b′*, small rubber tubes being used in the joints.

The parts *g g* and *g′g′* are blocks of hard rubber secured by the nuts *n* and *n′* to the brass tubes *t* and *t′*. The object of these blocks is to guide the flow of the water across the surfaces of the disk in order to make the temperature over each surface as nearly as possible everywhere the same.

The block *g′g′* is not fastened to the disk *d′d′* upon which it rests, but is carried by the tube *t′* in such a way that when *t′* is rotated *g′g′* turns with it. It was the practice at certain stages of the work to rotate *t′* briskly back and forth through a number of degrees, in order to facilitate the escape of air-bubbles from the space above and around *g′g′*. Numerous small holes drilled through the upper and outer edge of *g′g′* were intended to make, and probably did make, this operation more effective,

Most of the water (80 to 85%) coming up through *t′* passed down and out by way of the double funnel *f′f′*, but in order to remove the air-bubbles a certain amount of overflow was maintained through seven small tubes leading obliquely upward and outward from the water space at the edge of *g′g′*. One of these overflow tubes is shown in the figure at the left. Each of them was connected with a small rubber tube, and the outer end of each rubber tube was about 2 cm. higher than the top of *hh*. Whenever the tube *t′* and the block *g′g′* were being agitated to remove air-bubbles, as before described, the main outlet *o′* was closed, so that all the water which passed under the disk had to escape by the overflow tubes. This method of getting rid of air-bubbles was adopted after the problem had been studied by means of a kind of dummy apparatus, in which the disk *S* was replaced by a plate of glass. There is reason to believe that it was fairly effective, but it was cumbrous, and probably a different arrangement will be used in future work. No similar device appeared to be necessary for the space immediately above the disk, as air-bubbles originally there would naturally be carried upward through the tube *t* by the escaping water. A single overflow tube, *q*, led out from the double funnel *ff*, by which the water was brought to the upper surface of the disk, and through this tube a small overflow was maintained to carry off bubbles that might otherwise have caused trouble. Another function of *q* was to serve as a kind of gauge indicating roughly the amount of pressure exerted by the water within the apparatus, some parts of which were not well calculated to bear a great disruptive force.

The copper wire *w*, about 0.016 cm. in diameter, leads from a point on the upper copper coating of the disk through a hole in *h*, where it is held in a water-tight joint between a hard rubber peg and a soft rubber sheath surrounding the peg, to a wooden shelf, where its end is gripped between two copper washers under the head of a screw, all of which objects are shown in the figure. A thicker copper wire, about 0.06 cm., one end of which is pinched between the upper washer and the head of the screw, leads off toward a galvanometer. Another thin copper wire, *w'*, similar to *w*, starting from a corresponding point on the under copper coating of the disk, passes through *h'* and is fastened at its outer end exactly as the wire *w* is fastened. It is necessary to consider these details, for one can easily imagine a method of connecting the fine wires with the thick wires that would give rise to very disturbing thermo-electric forces and make the whole investigation useless.

At first the copper wires *w* and *w'* were *soldered* to the copper faces of the disk, as little solder as possible being used, with the hope that the thermo-electric forces at the top and bottom of the very thin layer of solder used would be very small in comparison with the large thermo-electric forces due to the copper-steel junctions. But this hope was not justified. The disturbance arising from these soldered junctions was very serious. The fact appears to be that each copper wire, exposing on the whole a surface of many square millimeters to the water, kept its point of attachment with the *solder* much warmer or much colder, as the case might be, than the same point would have been kept by direct contact with the water. The effect is similar to that observed when a soldered junction between two copper wires, one thick and one thin, is held a short distance beneath the surface of water differing considerably in temperature from the air above it. In such a case, represented by Figure 2, the submerged part of the thin wire, and also its point of attachment with the layer of solder, has very nearly the same temperature as the water; but the submerged part of the thick wire does not so nearly attain this temperature. The result is a difference of temperature between the two points of attachment with the solder, and a consequent thermo-electric effect, which an unwary observer might attribute to difference in the thermo-electric quality of the two wires. If

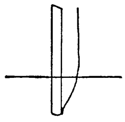
FIG. 2.

in such a case the junction is pushed deep beneath the surface, the observed effect is diminished and perhaps disappears.

Accordingly it was necessary to get rid of the soldered junctions upon the disk. Therefore the device was adopted of attaching the copper wires to the copper surfaces by means of copper electrolytically deposited. The details of the operation, where one junction only is to be dealt with, are as follows: Cover the surface of the disk, in the neighborhood of the spot where the junction is to be, with a thin layer of some non-conducting substance, paraffine for instance, and then scrape off this coating from a patch about 0.5 cm. long and 0.1 cm. wide. Coat also, with a non-conducting substance, the copper wire, and then scrape one end of it bare for a distance of about 1.0 cm. Lay the bare part of the wire along the middle of the bare strip on the copper disk and press it down flat in that position, the end of the wire projecting a little beyond the limits of the bare strip. While holding the wire firmly in place, attach it near each end of the bare strip by melting at each spot a bit of paraffine. A few drops of sulphate of copper * are now placed upon the bare part of wire and disk, above which they stand in a little mound, as in Figure 3. Another copper wire, the lower end of which dips a little distance into the mound of liquid, serves as the anode in the electrolysis. The exposed surfaces at the bottom of the liquid are the cathode, the current being carried off, not through the thin wire, but through the body of the disk, to another wire not here shown, the intensity of current aimed at being 0.02 ampere per sq. cm. of the exposed surface of the cathode. A run of three or four hours makes the junction sufficiently strong for proper use, though perhaps it is safer to let the action go on longer. After the deposit is completed the protruding end of the fine copper wire is cut off and the paraffine surrounding the junction is scraped away. When several junctions upon one side of the disk are to be formed, it is easy to make the deposit upon all of them simultaneously.

In the final use of the apparatus the copper wires must be kept from contact with the copper surface of the disk at other points than the legitimate junctions. Accordingly the wires and the disk were coated with a thin layer of asphaltum varnish, baked on at a temperature above that to which the apparatus was to be exposed in the conduction experiments.

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* Of specific gravity 1.10, with 20 drops of strong H_2SO_4 added to 200 cu. cm. of the solution.

There are thirteen of these fine wire junctions on each side of the disk, those on the upper side being arranged and numbered as shown by Figure 4. On the under side are corresponding junctions, 1′ just beneath 1, 2′ just beneath 2, 3′ just beneath 3, etc. The method of combining these junctions and the method of estimating the mean difference of temperature between the two copper-steel surfaces, by means of the thermo-electric currents sent through these junctions to a galvanometer, will be described later.

The parts j_1 and j_2, in Figure 1, are copper-German-silver junctions. The German-silver wire *g s*, about 0.015 cm. in diameter, is continuous from one junction to the other. At each junction the German-silver wire is joined to a copper wire, about 0.010 cm. in diameter. From each junction the two wires, separated by a strip of plastic non-conducting material ("Jenkin's packing"), *y*, are led into and along a plug, *p*, consisting of two half-cylinders of hard rubber held firmly together by clamps *k* and *k*. Within this plug the fine copper wire is attached in a soldered joint to another copper wire, *W*, about 0.06 cm. in diameter, which leads away toward a galvanometer. The loop of wire exposed to the water near each junction is coated with varnish to prevent disturbance due to chemical action. Shellac, baked on, seems best for this purpose. The water, as the arrows indicate, passes j_1 on entering, and j_2 on leaving, the upper chamber of the apparatus. The thermo-electric current from these junctions shows, when interpreted, the difference of temperature of the incoming and outgoing streams.

The amount of water flowing through the apparatus per second, about 24 grams, was measured by noting the time required to fill a flask of known capacity, *F*, in Figure 5. This time was about 75 seconds, and it was observed at stated intervals by means of an ordinary watch. The devices used for making this flow nearly constant, for an hour or more at a time, will be described later.

JJ, in Figure 1, is a double-walled copper jacket, supported by means of lugs, *l*, *l*, upon the ring *b b*, a thick piece of rubber under the end of each lug preventing metallic contact with *b*. The water, after passing the junction j_2 on its way out, entered this jacket near the bottom, flowed around and upward and escaped by the orifice *O* into a descending tube, the lower end of which was kept at any desired level, usually lower than any point shown in the figure. Thermometer T_1, passed by the stream on its way to j_1, and T_2, in the jacket near the exit, gave a rough indication of the change of temperature suffered by the water between these two points. It was usually about 1° C. when

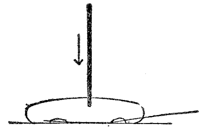

Fig. 3

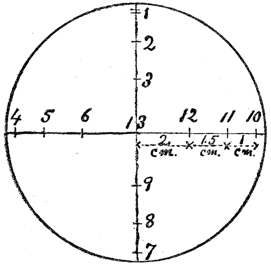

Fig. 4

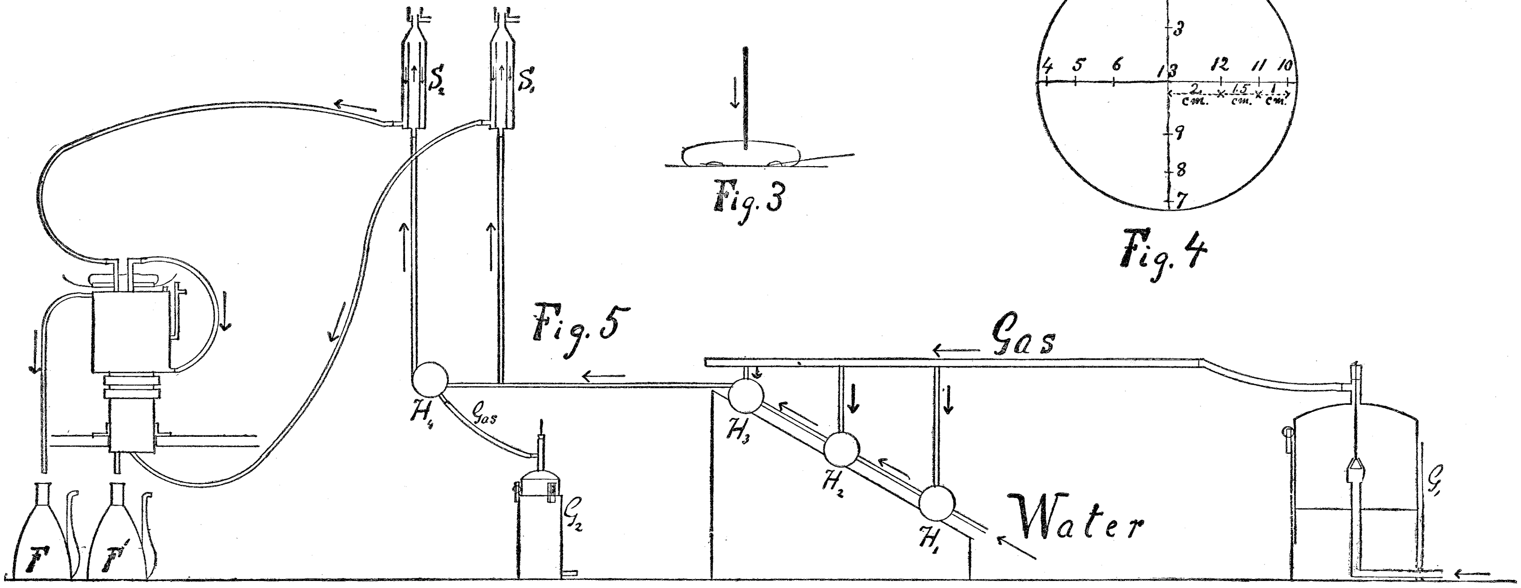

Fig. 5

it was a fall, and somewhat less when it was a rise, as one would expect from the fact that the jacket was usually warmer than the air surrounding it. The mean temperature of the jacket probably differed in most cases less than 1° C. from the temperature of the double funnel, *ff*, which it almost completely surrounded. The space around *h h* and *h′ h′* beneath the jacket, and the lower part of the jacket itself, was protected by a thick layer of cotton wool. The flat top of the jacket and the plugs and wires above it were protected in the same way. Under these conditions, the cooling or heating of the water by radiation or absorption at its envelopes between j_1 and j_2 was probably exceedingly small. As a further precaution against error from this source (and from certain other possible currents due to chemical action at j_1 and j_2, for instance), the stream passing through the upper chambers of the apparatus and over the upper surface of the disk was alternately made warmer and colder than the stream passing across the under surface of the disk.

The water-jacket *JJ* was open to the air at *A* to allow the escape of air-bubbles, which would otherwise have accumulated to such an extent as to affect the flow of water at *O*. Another outlet, at E, served as an overflow when the water came too fast to be delivered wholly through *O*. The streams from *O* and *E* were joined in the flask *F*, Figure 5. These minutiæ are of much importance, as the need of great uniformity in the rate of flow is imperative.

The general arrangement of the apparatus, with the exception of the electrical appliances, is shown in Figure 5. H_1, H_2, H_3, and H_4 are very effective contrivances * for heating a stream of water by the combustion of gas, which is supplied at constant pressure through two gasometers G_1 and G_2, one of which is shown in section. The water, which came from a large tank at the top of the Laboratory, with great regularity of flow when uninterrupted by obstructions in the apparatus, and at a nearly constant temperature, passed through the three heaters, H_1, H_2, and H_3 without division. Between H_3 and H_4 the stream divided, one part, about one half, going through H_4 to the standpipe S_2, the other part going without further heating to the standpipe S_1. These standpipes, from which a continual small overflow was maintained, had the double function of maintaining a constant head for the flow of the two streams through the conduction apparatus, and sifting out much of the air, which the heating had driven out of solution in the water. With the arrangement shown in Figure 5 the warmer stream of water enters the upper chamber of the conduction apparatus, and the cooler stream enters the lower chamber. This arrangement was, however, used alternately with another, in which the water entered S_1 after leaving H_4, the warmer stream going in this case to the lower chamber of the conduction apparatus. One stream was usually about 10° C. warmer than the other.

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* Made by the Buffalo Dental Manufacturing Company, No. 347 of their Catalogue for 1895.

It is now time to describe and discuss more fully the electrical devices and apparatus used for measuring, 1st, the difference in temperature of the two sides of the steel disk, and, 2d, the difference in temperature of the incoming and outgoing water of the upper chamber of the conduction apparatus.

CALIBRATION OF THE COPPER-STEEL THERMO-ELECTRIC ELEMENTS.

It has been stated that there were thirteen copper wires leading off from each copper coating of the steel disk. Each pair of wires, one above and the corresponding one below, represents a copper-steel thermo-electric element, consisting of a piece of steel about 0.3 cm. thick between two pieces of copper. To calibrate these elements *in situ*, that is, to determine by direct trial upon the disk the e. m. f. corresponding to a given difference of temperature between the upper and lower surfaces of the iron, is apparently impossible. Accordingly, two slender bars, one about 1 square millimeter in cross section, the other about one third as large in cross section, and each 7 or 8 cm. long, were cut from the same plate of steel from which the disk had been made, and tests were made with these,* copper wires having been soldered to the ends. The apparatus used for these tests is shown in Figure 6. B_1 and B_2 in this figure are vertical brass tubes through which streams of water at any desired temperature may be made to run. T_1 and T_2 are Baudin thermometers, remarkably similar in size and reading nearly alike at all temperatures, each graduated to 0°.2 C. The steel bars *b* and *b'*, and the copper wires soldered to them, extend through the rubber stoppers r_1 and r_2, so that the ends are exposed to the streams of water for a distance of about 1.3 cm. To prevent disturbance from chemical action upon the steel and copper, the parts exposed to the water had been dipped in a solution of copal in ether. With this protection of the junctions it appeared from tests made at various temperatures that there was no chemical action of sufficient magnitude to affect materially the results of the experiments made to determine their thermo-electric behavior. The tests for chemical action were made by exposing both ends of one bar to water at the same temperature, sometimes with the bar *in situ*, as in Figure 6, and sometimes not.

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* Experiments, in which a thin bar of steel, of the same quality as those here described, was subjected to longitudinal and to torsional stress, showed its thermo-electric quality to be very little affected by such conditions. Fearing, however, lest the *disk*, which had been subjected to certain processes which the thin bars had not suffered, might differ from them in thermo-electric quality, I have recently cut from the rim of a duplicate disk, made at the same time as the other, two narrow strips, which I have compared directly with the thinner of the two bars used in the copper-steel tests. These strips proved to be so like the bar in thermo-electric quality that it was difficult to make out with certainty any difference between them.

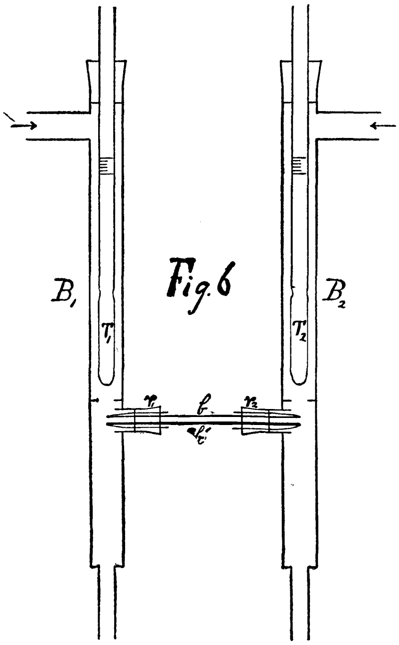

Fig. 6

The difference of temperature used in the thermo-electric tests was commonly about 4° C. The thermometers were read, by means of a telescope, to one tenth of one division, that is, to 0°.02. To eliminate disagreements of the thermometers, and various other possible sources of error, the streams were regularly interchanged, other conditions remaining the same. Thus, if a set of observations had been made with T_1 reading 28° and T_2 reading 24°, another would immediately be made with T_1 reading about 24° and T_2 about 28°.

The following course of reasoning will show how this method of using the thermometers tended to eliminate their errors from the result. Let us suppose that T_1 is used at 24° and T_2 at 28°, then T_1 at 28° and T_2 at 24°, and that afterward T_1 is used at 66° and T_2 at 70°, then T_1 at 70° and T_2 at 66°. Let $[E_1]_{20}$ be the error of T_1 at 20°, etc. Then we have in the various cases supposed: —

	True Difference of Temperature.	Error.
1st case	$28 + [E_2]_{28} - 24 + [E_1]_{24}$	$[E_2]_{28} - [E_1]_{24}$
2d case	$28 + [E_1]_{28} - 24 + [E_2]_{24}$	$[E_1]_{28} - [E_2]_{24}$

$$\text{Error of mean} = \tfrac{1}{2}\left([E_1]_{28} - [E_1]_{24}\right) + \tfrac{1}{2}\left([E_2]_{28} - [E_2]_{24}\right)$$

Similarly, the error of the mean at the higher temperatures is

$$\tfrac{1}{2}\left([E_1]_{70} - [E_1]_{66}\right) + \tfrac{1}{2}\left([E_2]_{70} - [E_2]_{66}\right).$$

If the "error of the mean" had been equally great at all observation stages on the thermometers, it would have affected the absolute value of the result obtained for the sensitiveness of the copper-steel thermo-electric element, but would have left unaffected the ratio of the sensitiveness at one temperature to that of any other temperature. Calibration of the thermometers showed that the "error of the mean" was not the same everywhere on the thermometers. The variation, though small, was perceptible, and might, if neglected, make the interval called 4° at one stage one half of one per cent greater or less than the interval called 4° at another stage. This error would not be very serious in view of the general character of the experiments if it had its full effect; but, in fact, the conclusion as to the conductivity of the steel depended upon the ratio of the sensitiveness of the copper-steel element at any temperature to that of the copper-German-silver element at or near the same temperature. Both kinds of element were calibrated by means of the same thermometers, and therefore the chance that final error of any magnitude could result from neglect of the errors of the thermometers is exceedingly small. Accordingly, such errors were neglected, the thermometers being used as if correct.

The question of course arises whether the two junctions differed in temperature just as much as the two thermometers did. It is no doubt true, that, when temperatures much above those of the room were used, each of the steel bars, several centimeters of which were exposed to the air, was a little cooler at each end than the water flowing past it. This difference was probably small, and must have been about the same at each end. The disturbance, or inaccuracy, caused by it should be smaller with the slender bar than with the other. Accordingly the slender bar was used most of the time, and all the results but one recorded in the following table were obtained with it. On the last day of the observations on copper-steel junctions one of the junctions on this bar broke. Then the thick bar was substituted for the other, and the result given for the temperature 74°.9 was thus obtained. This result agrees well enough with the others to indicate that there was no large difference in behavior of the two bars at the high temperatures. Previous trial had shown that they gave nearly equal effects at low temperatures. The one value here given for the thick bar was used with the others, obtained with the thin bar, in plotting the calibration line, Figure 7, for copper-steel junctions.

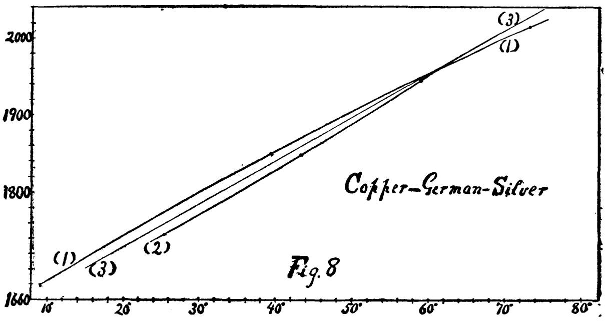

Fig. 8

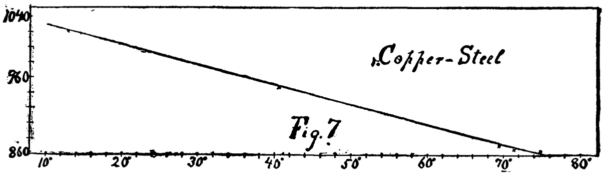

Fig. 7

The electric currents were measured by means of a fairly sensitive astatic galvanometer, the reduction factor of which was frequently determined by use of a current of known strength.

Copper-Steel Junctions.

Date.	Temperature of Room.	Mean Temperature of Junctions.	E. M. F. per 1° C. Diff. of Temp. of Junctions.
	°	°	
Sept. 25, 1894	21	22.6	994 × 10^{-8} volts
Nov. 9, "	12	12.7	1019 "
" 16, "	14	11.8	1019 "
Dec. 8, "	13	11.2	1034 "
Nov. 9, "	13	36.1	965 "
Dec. 8, "	13	40.6	949 "
Nov. 16, "	15	71.4	868 "
Dec. 8, "	13	69.5	873 "
" 12, (?) "	13.5	74.9	866 "

With these data the line given in Figure 7 was obtained. It is a straight line, the inclination of which indicates a neutral point in the neighborhood of 400° C.

Calibration of Copper-German-Silver Junctions.

It has already been stated that junctions of copper and German-silver were used to measure the difference of temperature of the incoming and outgoing water of the upper chamber. These junctions, held firmly in the plugs *p*, *p*, of Figure 1, were tested in the same apparatus and by the same method that had been used with the copper-steel junctions, the ends of the plugs fitting into the short side tubes occupied by rubber stoppers in Figure 6. The results presently to be recorded were obtained with two pairs of junctions called respectively No. 1 and No. 2. The same piece of german-silver wire was used in both pairs. The copper wires of No. 2 were not the same as those of No. 1, but all were taken from the same spool, that is, all had once been parts of one continuous wire.

The following method was sometimes used in making the junctions. The German-silver wire was heated in melted paraffine to the neighborhood of 230° C., the object being to forestall any change of its properties which might otherwise take place in the heat of soldering. In order to make sure that this temperature was not exceeded in the soldering, a bath of melted paraffine of about the same temperature was used to heat the soldering "iron." This device was abandoned after a time, experience seeming to show that it was unnecessary, and the junctions of Nos. 1 and 2, just mentioned, were not made in this way. The junctions of No. 1 were not coated with copal to protect them against chemical action of the water, but rosin was used in making this pair of junctions, and this left a partial coating which gave considerable protection. Experiments had appeared to show that a protective coating was not necessary. A test for disturbance due to chemical action with this pair of junctions was satisfactory, no evidence of such an effect being found. In making No. 2, however, the junctions were carefully coated with shellac, dried on at a temperature of about 80°. All of the conduction experiments of which the results will be given in this paper were made with No. 2, except those of August 13, which were made with a similar pair.

Copper-German-silver Junctions.

Junctions.	Date.	Mean Temperature of Junctions.		E. M. F. in Volts per 1° C. Diff. of Temp. of Junctions.	
No. 1	Dec. 27, 1894	9°.6	9°.2	1668 × 10^{-8}	1679 × 10^{-8}
"	" 28, "	9.6		1677 "	
"	Jan. 1, 1895	8.4		1691 "	
"	Dec. 28, 1894	39°.7	39°.5	1858 "	1848 "
"	Jan. 1, 1895	39.3		1838 "	
"	Dec. 28, 1894	73°.6	73°.4	2007 "	2015 "
"	Jan. 1, 1895	73.2		2022 "	
No. 2	Aug. 29, 1895	25°.4		1746 "	
"	" "	43.4		1849 "	
"	" "	59.1		1936 "	

August 31, 1895, further observations were made on No. 2 at 25°.4 and at 59°.2, but they were comparative only, as the reduction factor of the galvanometer was not determined on this day. If we *assume* the effect at 25°.4 to have been the same as on August 29, we get

No. 2	August 31	25°.4	1746
"	"	59.2	1951

It is found by interpolation that No. 1 would give for

25°.4	1772
43.4	1871
59.1	1949

The differences between the results obtained with the two pairs of junctions are probably within the limits of experimental error, and it appears better to use the results from both pairs in plotting the calibration curve than to use those from No. 2 only. No great accuracy is claimed for the absolute values obtained in any of the calibration tests. The calibration curve of No. 1 is shown as No. 1 in Figure 8. It is very slightly convex upward. The observations of August 31, reduced to absolute measure by means of the assumption already stated, are used, with those of August 29, to give data for the calibration curve of No. 2. This curve, No. 2 of Figure 8, is very slightly concave upward. Line No. 3, a mean between No. 1 and No. 2, is practically a straight line, which, if continued, would indicate a neutral point in the neighborhood of the absolute zero. In subsequent work this straight line is assumed to be correct, and data needed for use in the conductivity calculations are taken from it.

The Electric Circuits.

Figure 1 shows that the fine copper wires connected with the upper and under copper coatings of the steel disk led out to connections with heavier copper wires on a wooden shelf surrounding the apparatus carrying the disk. Some slight thermo-electric effect could be produced by warming one of these connections, but as the wires from both faces of the disk were connected here in the same way, the connections being all equidistant from the disk, the thermo-electric forces existing in them must have neutralized each other in the ordinary use of the apparatus. The copper wires leading away from these connections were all of the same length, size, and quality. They terminated in a kind of commutator, or switch-board, shown in Figure 9. The numerous small circles in this figure indicate small holes containing mercury. Holes 1, 2, 3, 4, etc. are connected, by means of the wires just mentioned, with junctions 1, 2, 3, 4, etc., respectively, on the disk. A connector, *C*, brings the four junctions 1, 4, 7, 10, which are (see Fig. 4) all at the same distance from the centre of the disk, into multiple arc, and takes the currents coming from them to one

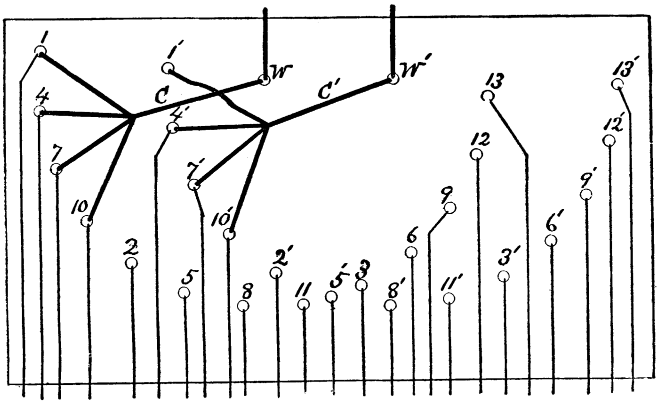

Fig. 9.

wire leading, through the mercury cup, *W*, to a galvanometer. Connector *C′* does the same for junctions 1′, 4′, 7′, and 10′. The same connectors can evidently be used to bring junctions 2, 5, 8, 11, and 2′, 5′, 8′, 11′ into action, or 3, 6, 9, 12, and 3′, 6′, 9′, 12′. The central junctions, 13 and 13′, were used by themselves.

The apparatus, beyond this switch-board, does not require detailed description. The resistance coils used were reduced to legal ohms by comparison with those of an Anthony standard (Queen & Co.). Thermometers at two or three points along the circuit gave such account of the temperature as was necessary. The galvanometer was a Thomson astatic, with a concave mirror. The strength of current required to produce a deflection of 1 cm. on the scale was about 12×10^{-8} amperes.

The circuit containing the copper-German-silver junctions, j_1 and j_2 of Figure 1, is simpler and requires little further description. The galvanometer of this circuit had a plane mirror, and was used with telescope and scale. Its sensitiveness to a given current was about the same as that of the first galvanometer. The circuit included, at a point corresponding to the switch-board of the other circuit, mercury cups at which the connection could be broken.

In addition to the mercury cups already mentioned, each circuit included a mercury commutator, by means of which the course of the current in the galvanometer could be reversed. These mercury cups and commutators were a source of much disturbance, and toward the last of the work solid metallic connections were in some cases substituted for them. The difficulty was thermo-electric. If two copper wires run into the same mercury cup at different levels, or if the two wires are of different size or different external condition, there will be at the copper-mercury junctions, unless the temperature of the room is more than usually uniform and constant, thermo-electric effects which cannot be neglected in delicate measurements of electric current. There was a certain approach to uniformity in these disturbances, which made it possible to eliminate them in great part from the result by the device, already described, of running the warmer stream of water alternately above and beneath the disk. The full measure of this source of error was, however, not realized until the last week or two of the work, and it may be that the discordance between the results of August 13 and 15 and those obtained later was due to lack of care in protecting the copper-mercury junctions from changes of temperature on the days mentioned.

Greater vigilance, and probably greater accuracy, marked the later work.

The difficulty here discussed can be very much reduced by a more sparing use of gas flames in the room.

The Observations.

As an example of the observations in the main experiment, showing their character and their arrangement, the last set made will now be given, some unimportant details being omitted.

The letters L and R refer merely to the position of the commutators. A change from L to R reverses the current in the galvanometer concerned. In each circuit the first observation is made under L, the second under R, the third under L, etc.

The note "1, 4, 7, 10," means that the junctions of the outer circle of the disk (see Fig. 4), both above and below, are in use, connected as in Figure 9. After this set of junctions, those of the next circle, 2, 5, 8, 11, are used, etc.

Each group of seven or five observations in the first circuit was made between two groups of three observations in the second circuit. Before the first, third, fifth, etc., groups of *three* in the second circuit, the main outlet of the water from the lower chamber of the apparatus was closed for a few seconds to wash out, through the overflow tubes (Fig. 1), the air-bubbles that might have accumulated beneath the disk. This operation did not greatly affect the current in the second circuit. The course of operations gave the first circuit about two minutes for recovery after the disturbance before observations in this circuit were renewed.

The water had been running through the apparatus for about an hour, as usual, before the observations began.

Junctions.	Copper-Steel Circuit.			Copper-German-silver.		
	L.	*R.*		*L.*	*R.*	
1, 4, 7, 10	36.28 36.20 36.14 36.15	30.76 30.80 30.80	5.40	46.98 46.90	40.85	6.09
				46.82 46.90	40.92	5.94
2, 5, 8, 11	35.80 35.78 35.76 35.78	31.04 31.04 31.06	4.73	46.94 46.90	40.85	6.07
				46.95 46.87	40.86	6.05
3, 6, 9, 12	35.15 35.14 35.10 35.11	31.58 31.58 31.61	3.54	46.84 46.86	40.78	6.07
				46.90 46.96	40.88	6.05
13	34.78 34.80 34.84	31.95 31.91	2.88	47.02 47.00	40.95	6.06
				46.85 46.91	41.00	5.88
3, 6, 9. 12	35.12 35.10 35.14 35.10	31.60 31.58 31.56	3.54	46.70 46.88	40.98	5.81
				46.75 46.81	41.10	5.68
2, 5, 8, 11	35.70 35.68 35.66 35.66	30.92 30.98 30.96	4.73	46.78 46.74	41.10	5.66
				46.80 46.70	41.14	5.61
1, 4, 7, 10	35.98 35.90 35.90 35.98	30.70 30.66 30.70	5.25	46.76 46.70	41.30	5.43
				46.65 46.65	41.18	5.47
					Mean,	5.85

Time occupied, 5:39 to 6:35 = 56 minutes.

The following observations accompanied those just recorded: —

t = temperature of water running from lower chamber.
t_1 = " " at thermometer T_1, Fig. 1.
t_2 = " " " T_2, "
S' = number of seconds required to fill flask F', Fig. 5.
S = " " " " F, "

The capacity of each flask was about 1800 grams. The numbers in brackets in the column headed S' indicate the number of grams of water escaping per minute through the overflow tubes of the lower chamber. In the column S is a similar note for the upper chamber.

Time.	t	t_1	t_2	S'	S	
5:33	22°.0	32°.8	31°.9			
5:35				70.5	72.5	72.8
				[300]	73.0	
6:02	22°.0	32°.5	31°.7		72.5	
6:37	21°.8	32°.2	31°.4			
6:40				71	73.5	73.5
				[290]	73.5	
					[50]	
Means	21°.9	32°.5	31°.7	70.8	72.9	

The 50 grams per minute noted as the overflow from the upper chamber is disregarded, not because it is inconsiderable in comparison with the main stream, which carried about 1400 grams per minute, but because it was taken off without having been brought near the disk, so that its temperature must have been practically unaffected by its admission to the chamber.

Assuming, for our present purpose, that no corrections need be applied to the thermometer readings, we see that the water escaping from the jacket is about 0°.8 colder than it was when it entered the upper chamber, that the difference of temperature of the streams in the two chambers is about 10°, and that the temperature of the disk must be about half way between those of the streams, 27°, let us say. The main object in noting the difference between t_1 and t_2 is to get information as to the effectiveness of the jacket that surrounds the upper chamber. If, as in this case, the temperature of the jacket differs only 1°, or thereabout, from the temperature of the chamber, the protection afforded must be nearly perfect. Apparently the difference was little, if any, more than 1° in any case.

The Calculations.

If the deflections obtained from the junctions at different points on the disk were all equal, or nearly so, the calculation of thermal conductivity from the observations would be exceedingly simple. Unequal as these deflections are, it would still be allowable to take their mean as representing the mean difference of temperature between the upper and lower surfaces of the steel disk, if all junctions represented equal areas of the disk; but this is not the case. The process of finding what the mean deflection would be, if all parts of the disk were equally represented, is indicated by the following formula, in which Δ stands for the mean deflection desired; δ for the deflection given by a particular pair of junctions; σ for the area of cross-section represented by this pair of junctions; S for the area of one face of the disk: —

$$\Delta = \frac{\Sigma\,\delta\,\sigma}{\Sigma\,\sigma} = \frac{\Sigma\,\delta\,\sigma}{S}.$$

If there were many pairs of junctions, but only one pair at a given distance, r, from the axis of the disk, each pair would represent a certain ring-shaped portion of the disk, the area of which at either surface is $2\,\pi\,r \times d\,r$. Writing this expression in place of σ, and calling the radius of the disk R, we get

$$\Delta = \frac{\Sigma\,\delta\,\sigma}{S} = \int_0^R \frac{\delta \times 2\,\pi\,r \times d\,r}{S} = \frac{2\,\pi}{S}\int_0^R \delta \times r \times d\,r.$$

We can approximate to this integration by means of a graphical process, making use of the data given by the observations actually made. From the series of observations given on page 290, we get the following values: —

r	δ
0	2.88
2 cm.	3.54
3.5 "	4.73
4.5 "	5.33 [$= \frac{1}{2}$ (5.40 + 5.25)]

The observed value of δ for $r = 0$ is to be increased a little to make allowance for the fact that the resistance of the circuit, including junctions 13 and 13′, was somewhat greater than the resistance of the circuit when the other junctions were in use. Making this slight change, and then taking the values of r as abscissas, and the values of δ as ordinates, we get the four determining points of curve A in Figure 10. Taking various ordinates from this curve, and multiplying each by the corresponding value of r, and, for convenience in plotting, by 0.2, we get the determining points of the curve a. The area under this curve is evidently

$$0.2\int_0^R \delta \times r \times dr,$$

from which the value of Δ is at once obtained. In this case $\Delta = 4.52$. The mean of the observed values of δ in the series under discussion is 4.12. All this is for the case where the water above the disk was warmer than that below the disk, on September 2.

The same day another series of observations was made, with the warmer stream beneath the disk. The curves B and b (Fig. 10) have to do with this second series of observations. They give for the value of Δ, 4.60.

It is assumed in the subsequent calculations that Δ is equal to the deflection which would be obtained from any pair of junctions on the disk, one above and one below, if the difference of temperature were made, for all values of r, equal to the difference between the mean temperature of the whole upper face and the mean temperature of the whole lower face of the steel disk. The validity of this assumption will be discussed later.

In the calculation presently to be made, the subscript 1 will have reference to the circuit containing the junctions on the disk; the subscript 2 will have reference to the circuit containing the copper-German-silver junctions.

The test for "sensitiveness" of the galvanometers, which has not been described at length, was made by running the same current through the two simultaneously. The "reduction factor," by which the deflection in centimeters must be multiplied to give the strength of this current in amperes, might be given for each galvanometer in numerical terms; but it will be evident that, for the calculation in hand, it is necessary to know only the ratio of the two reduction factors, which is the reciprocal of the observed simultaneous deflections of the two galvanometers in the sensitiveness test.

In finding the value of m, allowance is made for the change of capacity of the measuring flask with change of temperature of flask and water.

We have: —

k = the thermal conductivity in c. G. s. units.
Q = number of heat units flowing through disk in one second;
T = thickness, in centimeters, of the steel disk;
S = area, in square centimeters, of horizontal cross-section of the steel disk;
t = mean difference in temperature of sides of steel disk;
m = mass, in grams, of water traversing upper chamber per second;
Δ_1 = mean deflection in circuit 1, as before described;
Δ_2 = mean deflection in circuit 2, in conduction test;
R_1 = reduction factor of galvanometer 1;
R_2 = reduction factor of galvanometer 2;
d_1 = mean deflection of galvanometer 1 in sensitiveness test;
d_2 = mean deflection of galvanometer 2 in sensitiveness test;
$[R_1 \times d_1 = R_2 \times d_2$, and so $R_1 \div R_2 = d_2 \div d_1]$;
E_1 = e. m. f., in volts, of copper-steel element per 1° C. difference of temperature;
E_2 = e. m. f., in volts, of copper-German-silver element per 1° C. difference of temperature;
r_1 = resistance, in legal ohms, of circuit 1 in conduction test;
r_2 = resistance, in legal ohms, of circuit 2 in conduction test;

Then

$$k = \frac{Q \times T}{S \times t} = \frac{m \times \dfrac{\Delta_2 \times R_2 \times r_2}{E_2}}{S \times \dfrac{\Delta_1 \times R_1 \times r_1}{E_1}} \times T$$

$$= \left(m \times \frac{\Delta_2}{\Delta_1} \times \frac{R_2}{R_1} \times \frac{r_2}{r_1} \times \frac{E_1}{E_2}\right) \times \frac{T}{S}$$

$$= \left(m \times \frac{\Delta_2}{\Delta_1} \times \frac{d_1}{d_2} \times \frac{r_2}{r_1} \times \frac{E_1}{E_2}\right) \times \frac{T}{S}.$$

Using this formula with the data for series A and B of September 2, we have: —

A (warm water above.)			*B* (warm water below.)		
		log.			log.
$m = 24.57$		1.3904	$m = 24.58$		1.3906
$\Delta_2 = 5.85$		0.7672	$\Delta_2 = 4.91$		0.6911
$d_1 = 10.48$		1.0204	$d_1 = 10.48$		1.0204
$r_2 = 28.15$		1.4495	$r_2 = 28.14$		1.4493
$E_1 = 985 \times 10^{-8}$ [26°.9]		$\bar{6}.9934$	$E_1 = 985 \times 10^{-8}$ [27°.1]		$\bar{6}.9934$
	log.	$\bar{1}.6209$		log.	$\bar{1}.5448$
$\Delta_1 = 4.52$	0.6554		$\Delta_1 = 4.60$	0.6625	
$d_2 = 8.38$	0.9191		$d_2 = 8.30$	0.9191	
$r_1 = 16.58$	1.2196		$r_1 = 16.58$	1.2196	
$E_2 = 1796 \times 10^{-8}$ [32°.1]	$\bar{5}.2543$	$\bar{2}.0484$	$E_2 = 17.39 \times 10^{-8}$ [21°.8]	$\bar{5}.2403$	$\bar{2}.0415$
$\frac{T}{S} = \frac{0.2952}{77.9}$		1.5725			1.5033
		$\bar{3}.5786$			$\bar{3}.5786$
		$\bar{1}.1511$			$\bar{1}.0819$

$\bar{1}.1511$ = log. of 0.1416 = log. of k_a
$\bar{1}.0819$ = log. of 0.1208 = log. of k_b
Mean = 0.1312 = k

Thus, according to the observations of September 2, the conductivity of the steel at 27° C. is 0.1312.

The difference between k_a and k_b is large in this case, but it has already been explained that neither is intended to be complete by itself. The mean of the two is necessary to eliminate certain errors which have been discussed. It may well be doubted whether this elimination is perfect when the difference between k_a and k_b is so large. It will presently appear that the difference was exceptionally large on September 2.

The following table includes *all* the results obtained with the apparatus in its final condition. It has already been stated that the results of August 13 and August 15 do not agree with those found later, and a possible reason for this has been given, namely, lack of sufficient precaution, in the earlier experiments, against error from disturbing thermo-electric effects external to the conduction apparatus proper. Although the results obtained August 13 and 15 are here given, they are not included in finding the mean values given at the foot of each column. At one time it was suspected that an accumulation of dirt forming near the edge of the upper face of the disk, and checking the flow of heat there, might account for the discrepancy between the earlier and the later results. Accordingly, between August 28 and September 2 the conduction apparatus was dismounted, and the disk was cleaned and replaced. But the result obtained September 2 was in close agreement with most of the others.

Conductivity, *k*, of Open-Hearth Steel in C. G. S. Units.

Date.	Temperature.	k_a (Warm above.)	k_b (Warm below.)	k $\left(\frac{k_a+k_b}{2}\right)$.	
Aug. 13, 1895	[27°.0]	[0.1463]	[0.1445]	[0.1454]	0.1347
" 17, "	27°.4	0.1301	0.1392	0.1347	
" 20, "	27°.8	0.1333	0.1334	0.1334	
" 22, "	26°.3	0.1334	0.1310	0.1322	
" 28, "	27°.4	0.1306	0.1310	0.1308	
Sept. 2, "	27°.0	0.1416	0.1208	0.1312	
Mean,	27°.2	0.1338	0.1311	0.1325	
Aug. 24, 1895	44°.2	0.1299	0.1343	0.1321	
Aug. 15, "	[62°.8]	[0.1264]	[0.1463]	[0.1364]	0.1316
" 17, "	59°.8	0.1242	0.1347	0.1295	
" 22, "	59°.0	0.1301	0.1287	0.1294	
" 26, "	58°.8	0.1296	0.1327	0.1312	
Mean,	59°.2	0.1280	0.1320	0.1300	

Discussion and Criticism.

Taking 0.1325[*] as the value of *k* at 27°.2, and 0.1300 as the value at 59°.2, we get as the coefficient of change with rise of temperature

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[*] Winkelmann's *Handbuch der Physik*, 1895, gives : —

Metal.	Temp.	*k*	Observer.
Eisen	0°	0.1655	Lorenz.
"	0°	0.1988	Angström.
"	über 0°	0.1587	Berget.
"	ca. 15°	0.1648	Neumann.
"	ca. 15°	0.1133	Wiedemann u. Franz.
"	0°	0.1509	Mitchell.
"	0°	0.172	Stewart.
Schmiedeeisen	0°	0.2070	Forbes.
"	39°	0.1485	H. F. Weber.
Stahl hart	ca. 15°	0.062	F. Kohlrausch.
" weich	15°	0.111	"
" "	15°	0.1104	Wiedemann u. Franz.
Puddelstahl	15°	0.1375 – 0.1418	Kirchoff u. Hausemann.
Bessemer stahl	15°	0.0946	" "
Mangan stahl	0°	0.03280	Mitchell.

It is probable that open-hearth steel resembles the iron of this table more nearly than it does the steel.

$$\gamma = \frac{0.1300 - 0.1325}{0.1325 \times (59.2 - 27.2)} = -0.0006-.$$

If we take the values 0.1347 and 0.1316, obtained by including the bracketed values in finding the means, we get $\gamma = -0.0007+$. The value —0.0006 is to be preferred, although it is evident from the preceding pages that no great confidence can be placed in the digit of this coefficient. A wider range of temperature, a greater number of measurements, and evidently a more careful calibration of the copper-German-silver junctions is needed for a completely satisfactory determination of γ. Moreover, no allowance has here been made for the change in the specific heat of water between 27° and 59° C. It is to be observed that the lower temperature is very near the temperature of minimum specific heat of water. If it should be found that the specific heat of water at 59° is two per cent greater than at 27°, which is, to be sure, improbable, the value of γ obtained from these experiments would be 0. If the difference is one third of one per cent, which seems not unlikely, it will reduce γ to —0.0005. If we take this as the most probable value, it will, in spite of its uncertainty, be not devoid of interest, for the tables in Winkelmann give no values of γ in the case of steel, while for iron they give

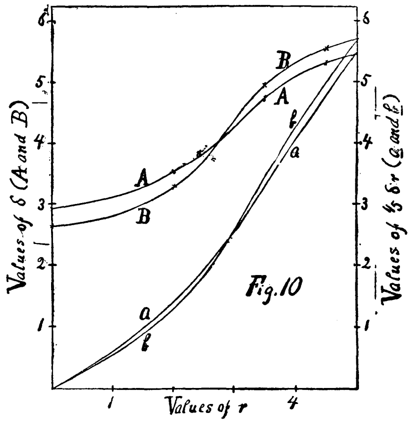
$\gamma = -0.000038$ (Lorenz),
$\gamma = -0.000517$ (Angström),
$\gamma = -0.0011$ (Stewart).

The one measurement of k for an intermediate temperature, 44°.2, lies between the mean value for 27°.2 and the mean for 59°.2, which is gratifying evidence, so far as it goes, of the precision of the measurements.

It is a curious fact that every measurement of k near 27°, except the last, gave a smaller value than the preceding measurement. The reason for this is not evident.

At the low temperature the mean of k_a is a little greater than the mean of k_b. At the high temperature the contrary is true. In either case the difference is to be regarded as mainly accidental, and not significant, — a fact already mentioned and explained.

We must now consider more fully the question whether Δ, as deduced from curves like those of Figure 10, represents accurately the difference between the mean temperatures of the upper and of the under face of the steel disk. It can hardly be doubted that Δ, derived as it is from thirteen well distributed pairs of junctions, represents with considerable accuracy the mean difference of potential between the two faces of the steel disk; but is this mean difference of potential proportional to the mean difference of temperature between these two faces?



This question would be at once answered in the affirmative if the disk, instead of being metallically continuous, were made up of little columns of steel, in length equal to the thickness of the disk, tipped at both ends with copper, each column being insulated from its neighbors. For in such an aggregation there would be no short-circuiting, and the difference of potential at the two ends of any steel column would depend merely upon the difference of temperature at these ends; and, with the very small difference of temperature existing, only a fraction of one degree, the potential difference would be very strictly proportional to the temperature difference.

But the disk is continuous, and there must be short-circuiting currents within it. How will these currents affect the mean difference of potential of the two faces? These currents in the steel will be partly vertical and partly horizontal. We will consider, first, the vertical components. Let Figure 11 represent any two equal vertical elements, S_1 and S_2, of the disk, connected by the copper strips C_1 and C_2. Let the e. m. f. of element S_1 be E_1, directed upward, and the e. m. f. of S_2 be E_2, also directed upward, and let $E_1 > E_2$. If connection between elements were broken, the difference of potential betweeen the top and bottom of S_1 would be E_1, and that between top and bottom of S_2 would be E_2. The mean difference of potential would be $\frac{1}{2}(E_1 + E_2)$.

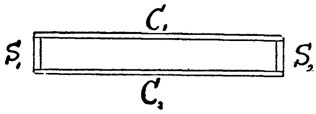


FIG. 11.

Let the resistance of S_1, equal to that of S_2, be called r. When connection exists, as in the figure, we have a short-circuiting current, C, passing up through S_1 and down through S_2. With this current, the difference of potential between top and bottom of S_1 is $E_1 - rC$, and the difference of potential between the top and bottom of S_2 is $E_2 + r C$. The mean of these two differences is $\frac{1}{2}(E_1 + E_2)$, the same value as if there were no short circuit.

If the element S_2, instead of being of the same size as S_1 had a cross-section n times as great, it would have n times as much weight as S_1 in reckoning the mean difference of potential, which would then be

$$\frac{1}{n+1}(E_1 + n E_2)$$

with open circuit, and

$$\frac{1}{n+1}\left[\left(E_1 - r C\right) + n\left(E_2 + \frac{r}{n} C\right)\right] = \frac{1}{n+1}\left(E_1 + n E_2\right)$$

with short circuit. This appears to show that the vertical components of short-circuiting currents within the steel disk do not affect the mean difference of potential between top and bottom.

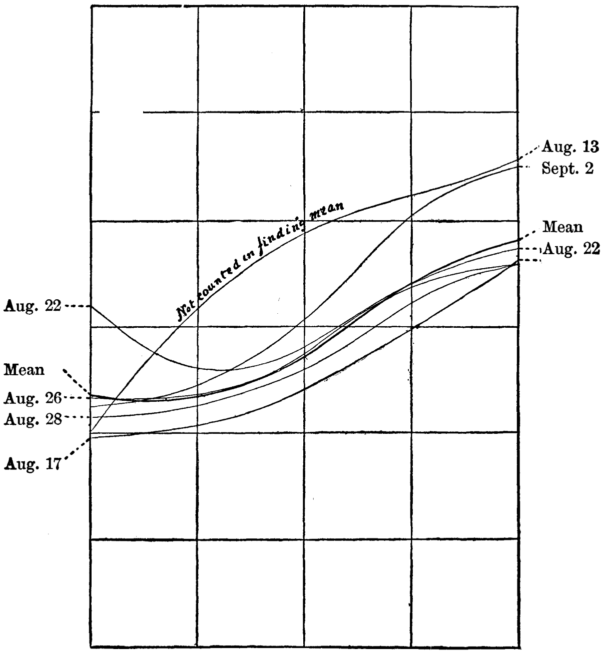


Fig. 12.

The non-effect of the horizontal components is shown by the consideration that a horizontal current, maintained in the disk by external or internal means, would have, of itself, no tendency to establish any mean difference of potential between the upper and under faces of the disk.

It appears, therefore, to be established that Δ does represent — that is, is proportional to — the mean difference of temperature between the upper and under faces of the disk.*

It may be of interest to compare the difference of potential curves of the various series of experiments. If in Figure 10 we take the mean of *A* and *B*, we get a new curve, which may be taken as representative of the potential conditions, or difference-of-potential conditions, prevailing in the experiments at low temperature on September 2. Combining each *A* curve for low temperatures with its corresponding *B* curve, and plotting the resultants, we get Figure 12, except the heavy line, which is obtained by taking the mean of all the others

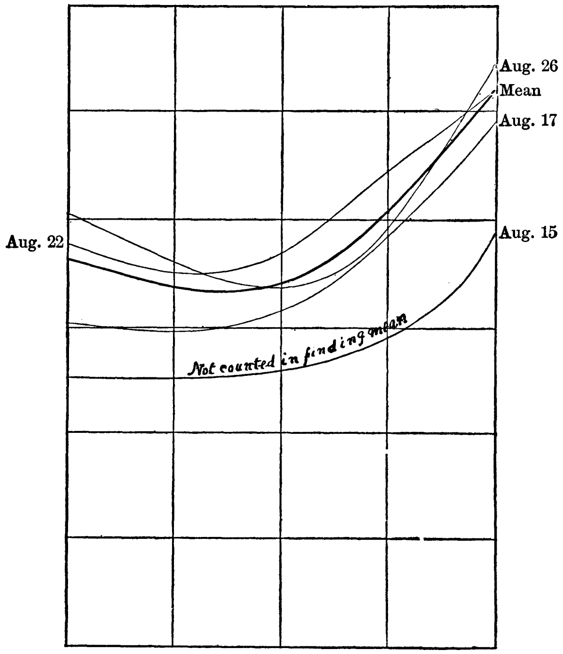


FIG. 13.

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* Cf. an article, by Prof. B. O. Peirce, "On the Properties of Batteries formed of Cells joined up in Multiple Arc." These Proceedings, Vol. XXX. p. 194.

except that of August 13. This heavy line is, therefore, the typical difference-of-potential curve for low temperatures, about 27°. The heavy line of Figure 13 is similarly the typical curve for high temperatures, near 59°, the curve for August 15 not being used in obtaining this composite.

The difference between these two typical curves is noticeable. Each indicates that the greatest difference of potential is near the edge of the disk. Each shows a minimum about two thirds as great as the maximum, but the minimum of the high temperature curve occurs farther from the centre of the disk. The reason for this difference is not obvious. The rapidity of the flow of water was about the same for high temperatures as for low, but there was a greater tendency to development of bubbles in the warmer water, and this may have influenced to some extent the course of the water over the surfaces of the disk.

The low temperature curve of September 2 is in marked contrast with most of the others in its group, showing a relatively great difference of potential at the edge of the disk. It has already been stated that the copper coatings of the disk had been cleared of accumulated dirt shortly before the experiments of September 2. This dirt had lodged mostly near the edge of the disk, and its removal apparently had just the effect that would have been expected upon the flow of heat in that region. The value of k obtained September 2 accords well with those obtained shortly before the disk was cleaned.

As to the absolute values found for k, they are very likely in error to the extent of two or three per cent of their own magnitude, possibly more. A source of possible error not yet discussed is found at the ring which encloses the disk (see Fig. 1). The effective diameter of the disk is taken to be the distance through the ring and disk, from the groove on one side to the groove on the other. The ring itself in this grooved part is about 0.08 cm. thick. Its cross-section is about three per cent of the whole conductive cross-section. The material of the ring is very similar to that of the disk, if not quite the same. The uncertainty as to whether the thin part of the ring can properly be assumed to act like an equal portion of the disk itself, comes from the dubious nature of the approach to this part of the ring above and below. The flow into or from it may be partly lateral, through the edge of the disk proper, but it cannot be wholly so, for a narrow strip of the thicker portion of the ring is exposed to the water above and below the disk. It is not likely that the error here is large, but in future experiments a different method of supporting the disks will probably be used.

It may be asked whether the method of investigation which has been described at such length in this paper will be applicable to the case of metals in general, and whether it can be extended through any large range of temperature. It is evident that the thermo-electric device for determining the difference of temperature between the two faces of the disk must be used with great caution, and will in some cases be difficult of operation; but its use is by no means restricted to iron. Copper and other metals thermo-electrically near it will certainly give some trouble, though it is believed that they can be successfully dealt with. It is probable that the range of temperature can be greatly increased by the use of other liquids instead of water; but if this is done, the question as to the specific heat of the liquid will become a difficult one.

The method described is a laborious one in its preparatory stages, but much of the initial work can be done by a mechanic. The operation of the apparatus, once prepared, is comparatively easy, and the calculations are simple. A single experimenter operates the whole machinery, and makes all the observations. He can make the A and B observations for a given temperature and calculate the value of k therefrom in one day, without excessive toil, and with a reasonable assurance that the result will be in accord with that of the day before.

This investigation was begun, at the author's suggestion, some years ago by Mr. A. W. Slocum, then a graduate student at Harvard, now of the University of Vermont. Mr. Slocum worked with great zeal and energy, and made substantial progress, but presently went abroad, leaving the research in my hands.

I have to thank all members of the Physical Department at Harvard, who have given me extremely valuable suggestions in regard to my work, and have furthered it by every means in their power.